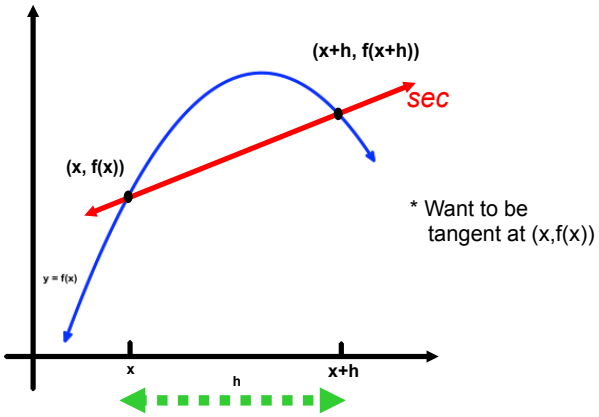
Derivatives

- *Finding the tangent to the graph of f(x) at a given point*



$$*slope = \frac{f(x+h)-f(x)}{x+h-x} = \frac{f(x+h)-f(x)}{h} = slope_{\sec}$$

$$*slope_{\tan} = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

* **Slope of a Curve**: *the slope of a curve at a given point is the same as the slope of the line that is tangent to the curve at that point.*

Ex) Find the tangent to the graph of $f(x) = x^2 + 3$ at (2,7).

To Solve: 1) Get point, if needed
2) Get Slope
3) Use Point-Slope formula

Ex) Find the tangent to the graph of $f(x) = x^2 + 4x$ at $x = 1$.

Ex) Write the equation of the line tangent to $f(x) = x^2 + x - 1$ at $x = 2$.

Ex) Write the equation of the line tangent to $f(x) = x^3$ at $x = -1$.

Definition of the Derivative

Def: If *f* is a function of x, the derivative of *f* with respect to x is the function *f* ' , when

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Notation: $\frac{df(x)}{dx}$ $f'(x)$ $\frac{dy}{dx}$ y'

Derivative at a Point: $\left.\frac{dy}{dx}\right|_{x=x_0} = f'(x_0)$

Applications Later !!!!!

Ex) Compute $f'(x)$ given $f(x) = 3x^2 + 2x$

Ex) a) Compute $f'(x)$ given $f(x) = \frac{1}{x}$

b) Find $f'(4)$

Ex) Given $y = \sqrt{2x+1}$

a) find $\frac{dy}{dx}$

b) Find the slope of the line tangent to y at x = 4.

c) Write the equation of the line tangent to the curve at x = 4.

Ex) Given $f(x) = 4x^2 - 3x$, find $f'(2)$.

Differentiation Rules

1. **Derivative of a Constant**
   - if c is a constant, then

$$\frac{d}{dx}(c) = 0$$

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2. **Power Rule**

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

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3. **Sum and Difference Rule**

$$\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$$

Examples)

1.) $\frac{d}{dx}(x^2) =$

2.) $\frac{d}{dx}(5x^4)$

3.) $y = x^4 + x^2 - x^{-1}$

$y' =$

4.) $f(x) = \sqrt{x}$

$f'(x) =$

5.) $f(x) = \frac{x^2 - 3x}{x}$

$f'(x) =$

6.) $y = \frac{5}{x^2}$

$y' =$

The Product Rule

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

Ex 1) $f(x) = (4x^2)(3x^5)$ $f'(x)=$

Ex 2) $f(x) = (x^2+1)(2x+4)$ $f'(x) =$

Ex 3) $y = (x^2+x)^2$ $y' =$

The Quotient Rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Ex 1) $f(x) = \frac{12x^5}{2x^3}$ $f'(x) =$

Ex 2) $f(x) = \frac{x^2+1}{3x}$ $f'(x) =$

Ex 3) $y = \dfrac{1+x^2}{1-x^2}$ $\qquad \dfrac{dy}{dx} =$

Ex 4) $y = \dfrac{4x+1}{x^2-5}$ $\qquad \left.\dfrac{dy}{dx}\right|_{x=1}$

Ex 5) $f(x) = (x+1)\left(\dfrac{x^2+2}{x-1}\right)$

Higher Order Derivatives

If the derivative f' of a function f is itself differentiable, then the derivative of f' is denoted by f'', called the second derivative. If the result is differentiable, we can continue!

$$f' = \frac{dy}{dx} = y'$$ 1st Derivative

$$f'' = \frac{d^2y}{dx^2} = y''$$ 2nd Derivative

$$f''' = \frac{d^3y}{dx^3} = y'''$$ 3rd Derivative

, etc...

Ex 1) Find all the derivatives of:

$$f(x) = 3x^4 - 2x^3 + x^2 - 4x + 2$$

Ex 2) $Find \ \frac{d^2y}{dx^2} \ if \ \ y = 7x^3 - 5x^2 + x$

Ex 3) *Find* $f''(1)$ *if* $f(x) = \sqrt{x} - \frac{1}{x^2}$

Rule Check:

Suppose f and g are functions of x, and f and g are differentiable at x = 0, if f(0) = 5, f'(0) = -3, g(0) = -1, and g'(0) = 2.

Evaluate the following:

1) $\frac{d}{dx}(fg) =$

2) $\frac{d}{dx}\left(\frac{f}{g}\right) =$

3) $\frac{d}{dx}(7f - 2g)$

The Chain Rule

- until now, we have been only taking the derivative of powers of x. This rule will allow us to take the derivative of any differentiable function raised to a power.

$$\frac{d}{dx}u^n = n \cdot u^{n-1} \cdot \frac{du}{dx}$$

Note: u = any differentiable function n = any real number

Ex) $y = (4x-1)^2$ $u = 4x-1$ $n = 2$

$y' = 2(4x-1)^1(4) = 8(4x-1) = 32x-8$

Ex) $y = (x^2 + 5x)^{11}$ $u = x^2 + 5x$ $n = 11$

$y' =$

The Chain Rule

- until now, we have been only taking the derivative of powers of x. This rule will allow us to take the derivative of any differentiable function raised to a power.

$$\frac{d}{dx}u^n = n \cdot u^{n-1} \cdot \frac{du}{dx}$$

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Ex) $y = (4x-1)^2$ $u = 4x-1$ $n = 2$

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Ex) $y = (x^2 + 5x)^{11}$ $u = x^2 + 5x$ $n = 11$

$y' =$

Ex) $y = \sqrt{2x-1}$ Find y'

Ex) $f(x) = (x+3)^2(x-1)^3$ Find $f'(x)$

Ex) $y = \dfrac{(x-3)^4}{x^2+2x}$ Find y'

**** Chain Rule Check***

x	f(x)	f '(x)	g(x)	g '(x)
3	5	-2	5	7
5	3	-1	12	4

a) Find $F'(3)$ where $F(x) = f(g(x))$

b) Find $G'(3)$ where $G(x) = g(f(x))$

c) Find $G'(5)$ where $G(x) = (g(x))^2 + f(x)g(x)$

Ex) If $f(2) = 3$ and $f'(4) = 1$, $f'(2) = -3$

Evaluate $F'(2)$ if....

$F(x) = f(x^2) + f(x)$

Trigonometric Derivatives

* TO BE MEMORIZED!!!

1. $\frac{d}{dx}\sin u = \cos u \cdot \frac{du}{dx}$	2. $\frac{d}{dx}\cos u = -\sin u \cdot \frac{du}{dx}$
3. $\frac{d}{dx}\tan u = \sec^2 u \cdot \frac{du}{dx}$	4. $\frac{d}{dx}\sec u = \sec u \cdot \tan u \cdot \frac{du}{dx}$
5. $\frac{d}{dx}\cot u = -\csc^2 u \cdot \frac{du}{dx}$	6. $\frac{d}{dx}\csc u = -\csc u \cdot \cot u \frac{du}{dx}$

Ex 1) $y = \sin x$

$y' =$

Ex 2) $y = 6\cot x$

$y' =$

Ex 3) $y = \tan(2x)$

$y' =$

Ex 4) $y = \cos(x^2 - 5)$

$y' =$

Ex 5) $y = x^2 \sin x$

$y' =$

Ex 6) $y = \dfrac{\cos x}{\sin x + 1}$

$y' =$

Ex 7) $y = \sin^3 x$

$y' =$

Ex 8) $y = \cos^2(3x^4)$

$y' =$

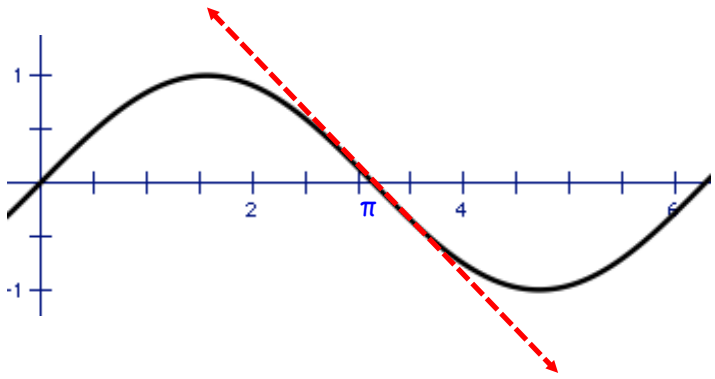
Ex 9) $y = \tan(\cos x)$

$y' =$

Ex 10) $y = \cos(\sin 3x^2)$

$y' =$

Ex 11) Find the equation of the line tangent to the graph of $y = \sin x$ at $x = \pi$.



Finally.....What if....this was the question.....

1. $\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$ =

2. $\lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h}$ =