Curve Sketching w/ Derivatives

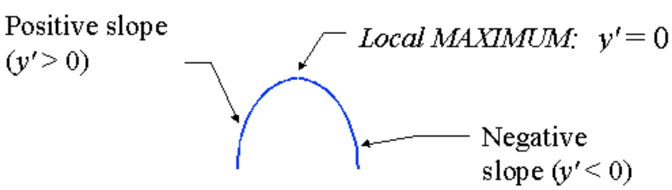
The kinds of things we will be searching for in this section are:

x-intercepts	Use $y = 0$ NOTE: In many cases, finding x-intercepts is not so easy. If so, delete this step.
y-intercepts	Use $x = 0$
local maxima	Use $dy/dx = 0$, sign: $+ \rightarrow -$
local minima	Use $dy/dx = 0$, sign: $- \rightarrow +$
points of inflection	Use $d^2y/dx^2 = 0$, **and** sign of d^2y/dx^2 changes

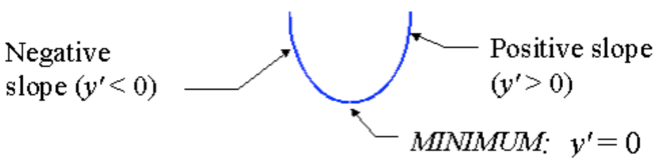
Finding Maxima and Minima

Extrema - Relative/Local and Absolute

A **local maximum** occurs when $y' = 0$ and y' changes sign from positive to negative (as we go left to right).



A **local minimum** occurs when $y' = 0$ and y' changes sign from negative to positive.



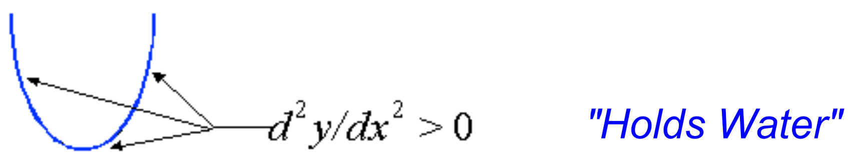
Finding Absolute Extrema of *f(x)* on [*a*,*b*].

0. Verify that the function is continuous on the interval [*a*,*b*].
1. Find all critical points of *f(x)* that are in the interval [*a*,*b*]. This makes sense if you think about it. Since we are only interested in what the function is doing in this interval we don't care about critical points that fall outside the interval.
2. Evaluate the function at the critical points found in step 1 and the end points.
3. Identify the absolute extrema.

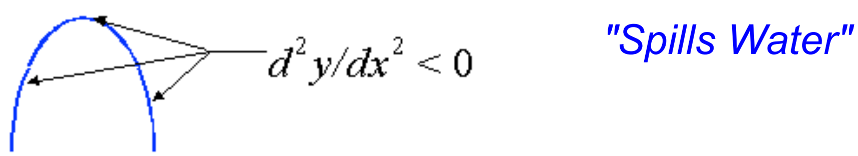
The Second Derivative

The second derivative can tell us the *shape* of a curve at any point.

- If $d^2y/dx^2 > 0$, the curve will have a **minimum**-type shape (called *concave up*)



- If $d^2y/dx^2 < 0$, the curve will have a **maximum**-type shape (called *concave down*)



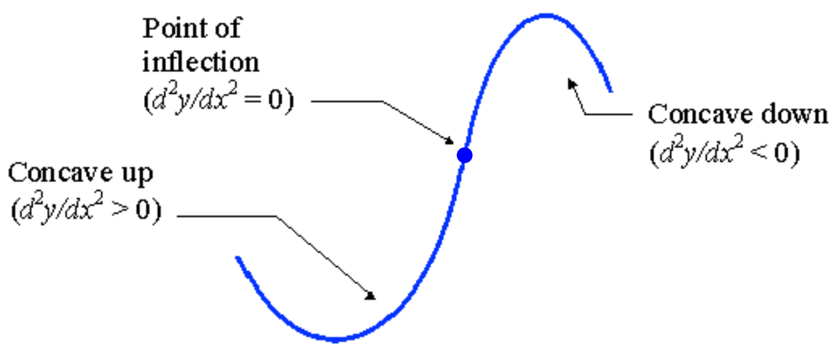
Finding Points of Inflection

A point of inflection is a point where the shape of the curve changes from a **maximum**-type shape ($d^2y/dx^2 < 0$) to a **minimum**-type shape ($d^2y/dx^2 > 0$).

Clearly, the point of inflection will occur when

$d^2y/dx^2 = 0$ **and** when there is a change in sign

(from plus → minus or minus → plus) of d^2y/dx^2.



* Steps: Full Graph Analysis

1. First Derivative:

Max/Min:
- Set f'(x) = 0
- Solve (for possible max/min)
- Number line Analysis
  - * Max ⇨ + to -
  - * Min ⇨ - to +

Increasing/Decreasing:

if f'(x) > 0	INCREASING
if f'(x) < 0	DECREASING
if f'(x) = 0	CONSTANT

2. Second Derivative:

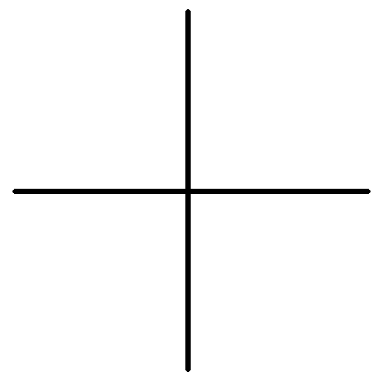
Points of Inflection:
- Set f"(x) = 0
- Solve (for possible POI's)
- Number line Analysis
  - * POI's occur with sigh change

Concavity:

if f"(x) > 0	Concave UP
if f"(x) < 0	Concave DOWN

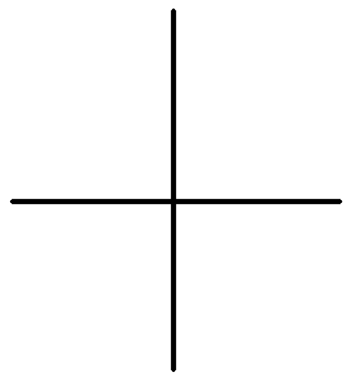
Ex) Given $y = x^3 - 6x^2 + 9x + 1$

Find:
- *All Possible Max/Min's*
- *Intervals of Inc/Dec*
- *All Possible POI's*
- *Intervals of Concave Up/Down*
- *Sketch*



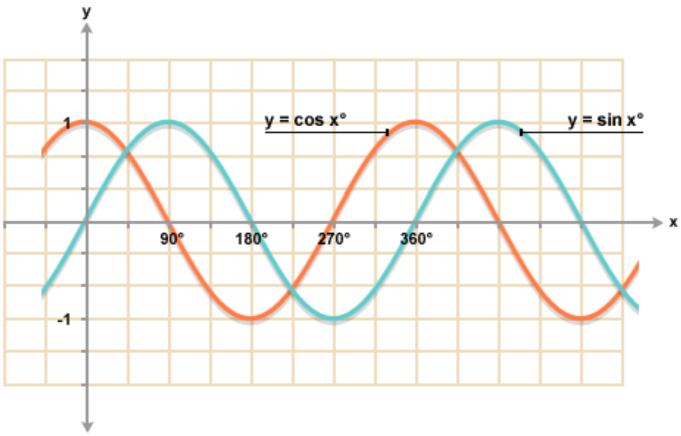
Ex) Given $y = 3x^4 + 4x^3 + 1$

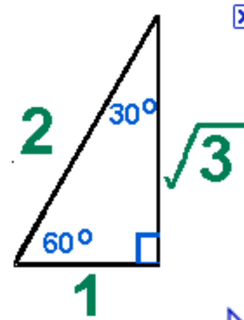
Find:
- *All Possible Max/Min's*
- *Intervals of Inc/Dec*
- *All Possible POI's*
- *Intervals of Concave Up/Down*
- *Sketch*

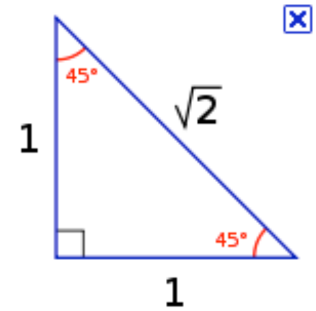


Graphing Trig Functions

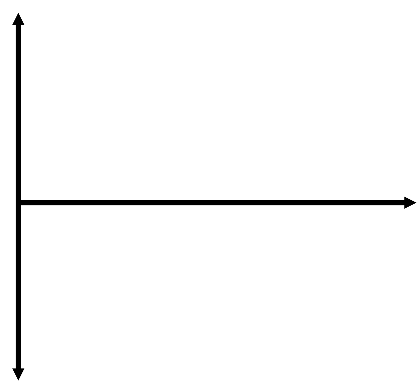
Steps: 1. ALL Previous Steps
2. Know Trig Values!



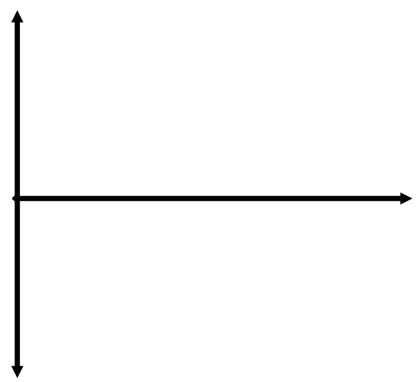




Ex) $y = x - \sin x$ $[0, 2\pi]$



Ex) $y = x + \cos x$ $[0, 2\pi]$



Graphing Rational Functions

- Remember, Polynomial Functions are smooth, continuous and differentiable!
- Rational functions will be discontinuous!

Steps: 1. Find ALL Horizontal and Vertical Asymptotes.
2. All previous Steps.

Ex) Complete a full graph analysis of $y = \dfrac{x-1}{x-2}$

Ex) Complete a full graph analysis of $y = \dfrac{x^2}{x^2 - 1}$

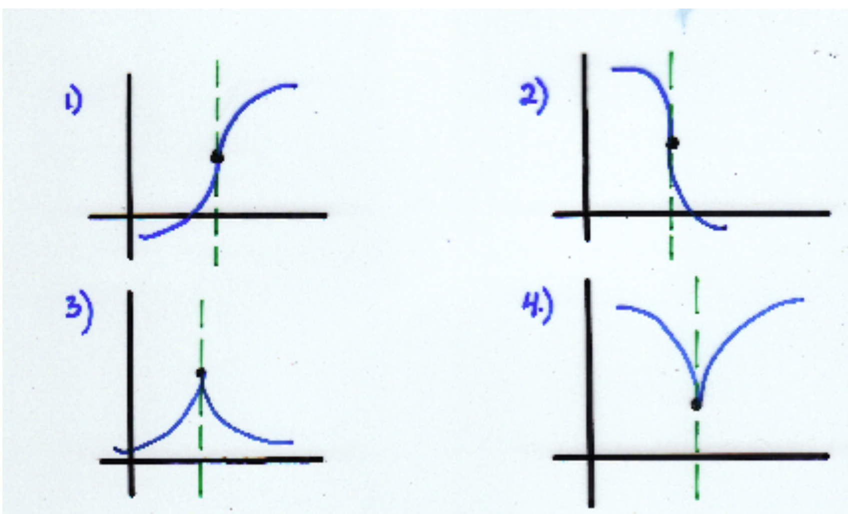
More Graphing Characteristics

• *Vertical Tangents*

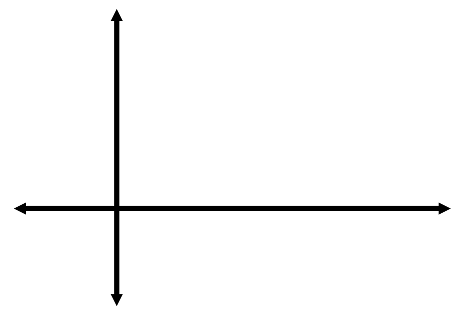
A graph has a vertical tangent at x_o if ;

1. *f(x) is continuous at x_o*
2. *f'(x) is UNDEFINED at x_o*

Four Types:



Ex) Complete a full graph analysis of $f(x) = (1 - 3x)^{2/3}$



Translating f ' to f

Example 1. Sketch a graph of the **y = f(x)** given the sketch of its derivative shown in Figure 1.

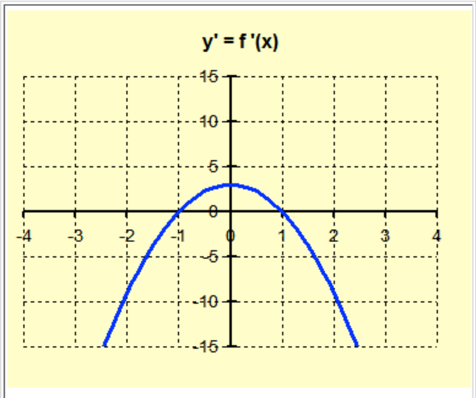
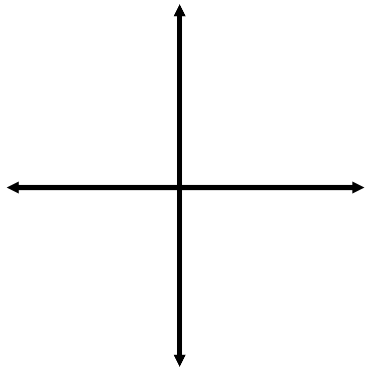


Figure 1.

**** Given, f(0) = 1***



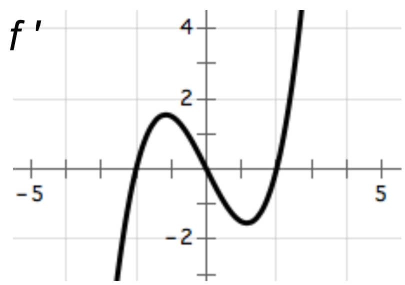
To create a possible sketch of the original given the derivative;

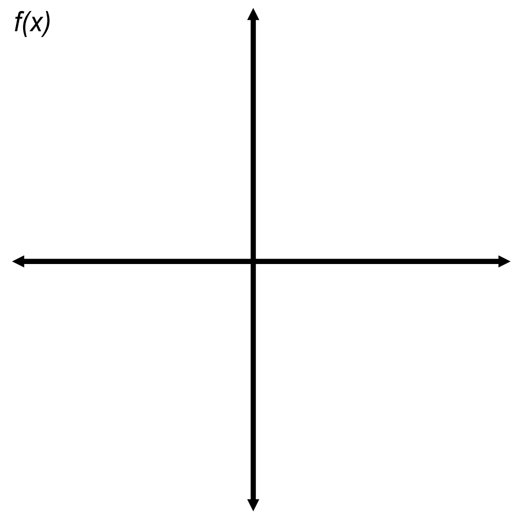
1. f '(x) Number line: - *f '* zeroes (*Max/Min/Horiz. Tans of f*)
   - *f '* Above (*f Increasing*)
   - *f '* Below (*f Decreasing*)

2. f "(x) Number Line: - Max/Min of f ' (*POI's of f*)
   - *f"* Increasing (*f Concave UP*)
   - *f"* Decreasing (*f Concave DOWN*)

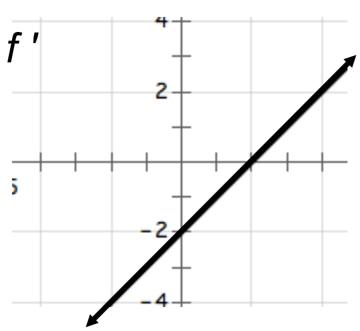
* *Watch for a given point, We must satisfy this given condition !*

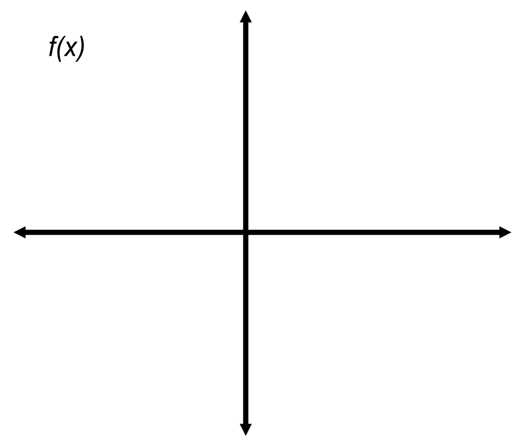
Ex) Given f ' below, Sketch a possible solution of f with f(1) = 0.



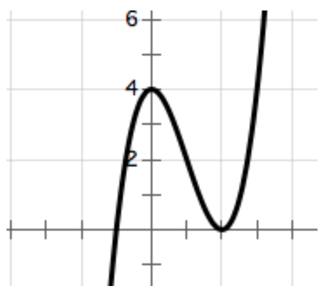


Ex) Given f ' below, sketch a possible solution of f is f(0) = 3.

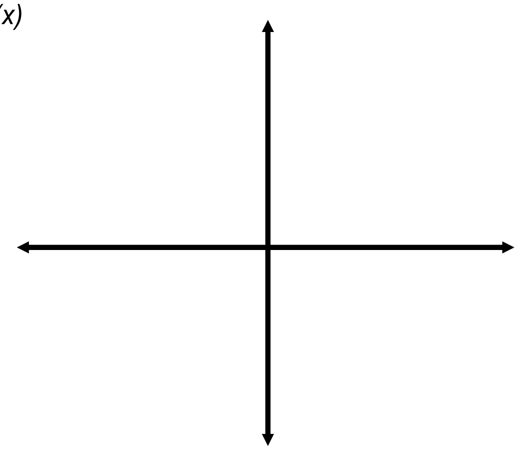




Ex) Given f ' below, Sketch a possible solution of f with f(-1) = 0.



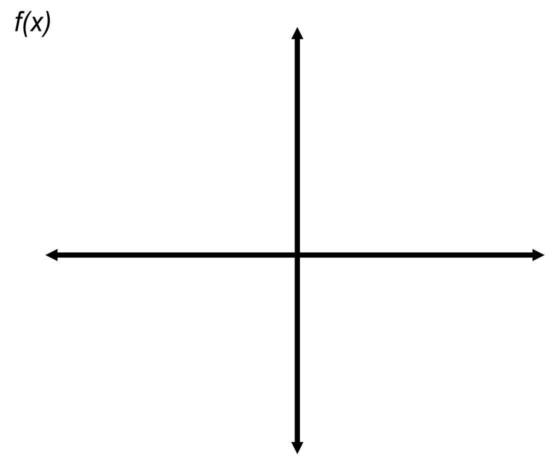
f(x)



• Using Sign Charts to Sketch f

• Sketch a possible graph of *f* given the table below, with *f(0) = -2* and *f* is ***continuous***.

Interval	*f'(x)*	*f''(x)*
x < 1	+	+
1 < x < 3	+	-
x > 3	+	+



* f is a continuous function on [-4,3] with f(-4) = 6, f(3) = 2, and the following properties;

Interval	*(-4,-2)*	*x = -2*	*(-2,1)*	*x = 1*	*(1,3)*
f'	-	0	-	*und.*	+
f''	+	0	-	*und.*	-