Chapter 1 – Positive Numbers and the Number Line

1.1 – The Number Line

whole number - any of the numbers 0, 1, 2, 3, 4 and so on

positive number - a number that is greater than zero.

- Positive numbers also include decimals and fractions.

horizontal number line - lesser number always lies to the left of greater number

vertical number line - lesser number always lies below the greater number

Learn **Represent numbers on a number line.**

Label each number line horizontal or vertical.

vertical

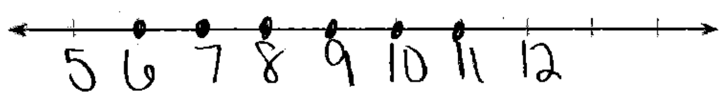
add points

On the horizontal line represent the whole numbers between 5 and 12.

add points

On the vertical line represent the whole numbers from 5 to 12.

horizontal



You can use a number line to compare whole numbers.

For example, in the number line shown, 35 lies to the right of 33.

So, 35 is greater than 33.

This can be represented by 35 > 33.

Guided Practice

add points

Draw a horizontal number line to represent each set of whole numbers.

1. Positive whole numbers less than 5

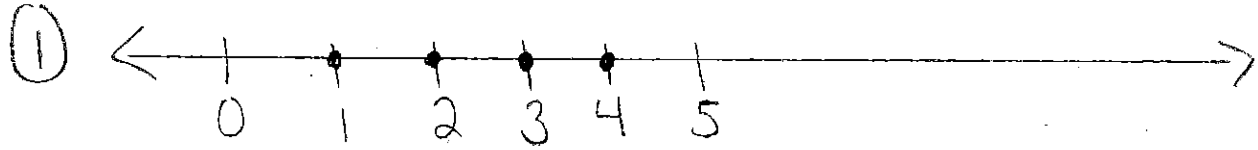
2. Whole numbers greater than 9 but less than 14

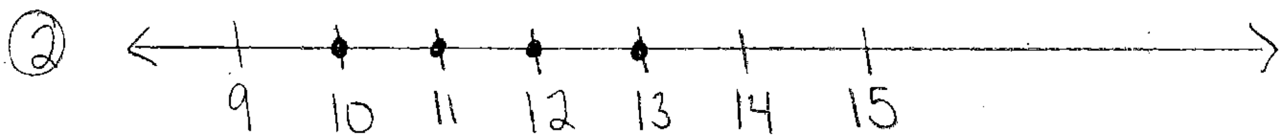
add points

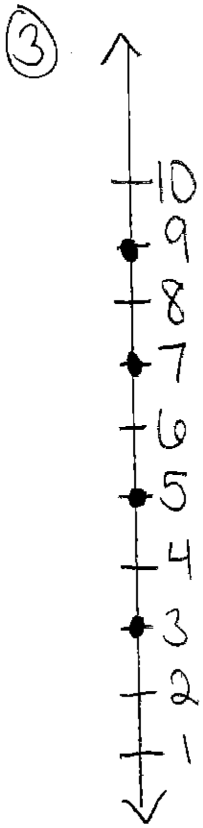
Draw a vertical number line to represent each set of whole numbers.

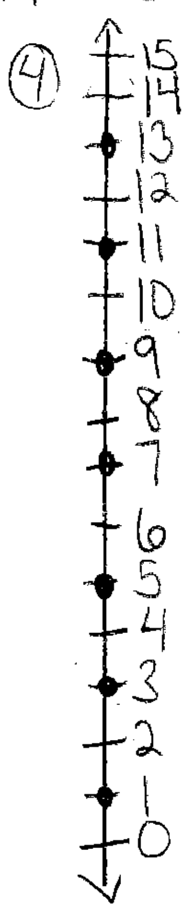
3. Odd numbers between 1 and 10

4. Positive odd numbers < 15





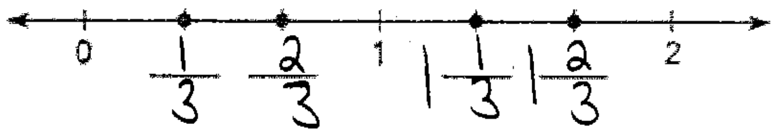




Guided Practice

Complete each ____ with the correct value, and each $\square$ with > or <.

5 Fill in the missing fractions and mixed numbers on the number line. Then complete the statements of inequality.



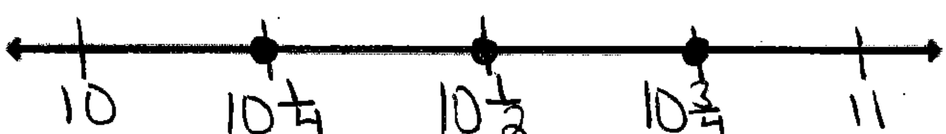
$\frac{2}{3} \boxed{<} 1$

$2 \boxed{>} 1\frac{1}{3}$

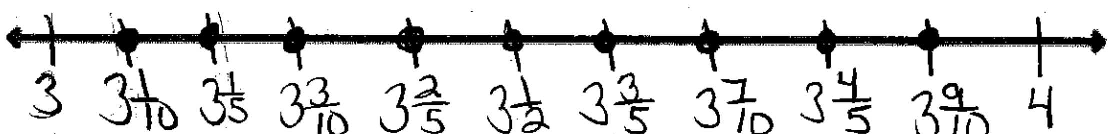
Draw a horizontal number line to represent each set of numbers.

6 Mixed numbers greater than 10 but less than 11

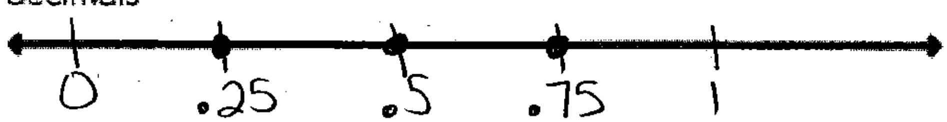
Use an interval of $\frac{1}{4}$ between each pair of mixed numbers.



7 Mixed numbers from 3 to 4, with an interval of $\frac{1}{10}$ between each pair of mixed numbers

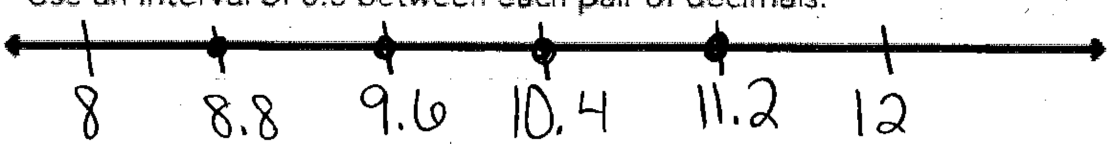


8 Decimals between 0 and 1, with an interval of 0.25 between each pair of decimals



9 Decimals greater than 8.0 but less than 12.0

Use an interval of 0.8 between each pair of decimals.



Guided Practice

Complete each ____ with the correct value, and each ☐ with > or <.

10 a) Fill in the missing decimals on the number line.

b) Compare each pair of decimals using < or >. Use the number line in to help you.

0.1 $\boxed{>}$ 0.05 0.02 $\boxed{<}$ 0.07

Draw a vertical number line to represent each set of numbers.

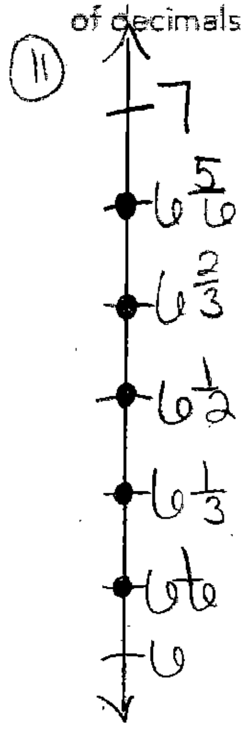
11 Mixed numbers greater than 6 but less than 7
Use an interval of $\frac{1}{6}$ between each pair of mixed numbers.

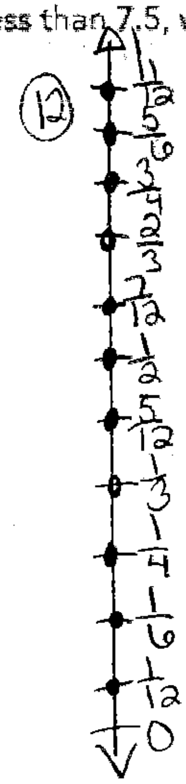
12 Positive fractions less than 1, with an interval of $\frac{1}{12}$ between each pair of fractions

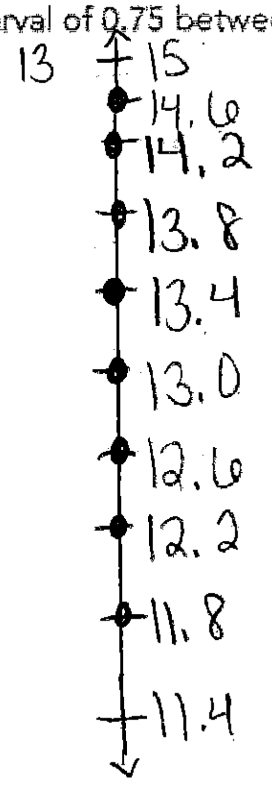
13 Decimals between 11.4 and 15.0, with an interval of 0.4 between each pair of decimals

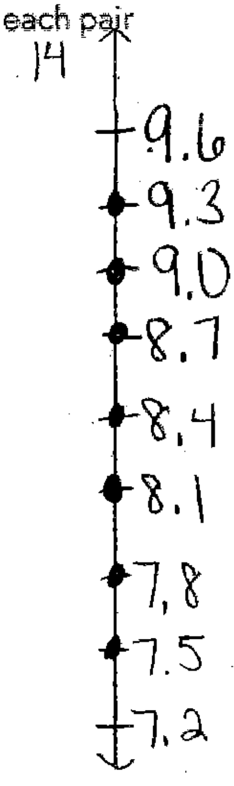
14 Decimals greater than 7.2 but less than 9.6
Use an interval of 0.3 between each pair of decimals.

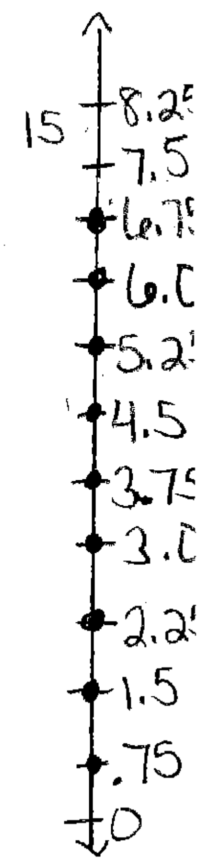
15 Positive decimals less than 7.5, with an interval of 0.75 between each pair of decimals











Compare each pair of numbers using > or <. Use a number line to help you.

16. $3\frac{9}{10} \boxed{>} 3\frac{3}{10}$

17. $2.17 \boxed{<} 2.71$

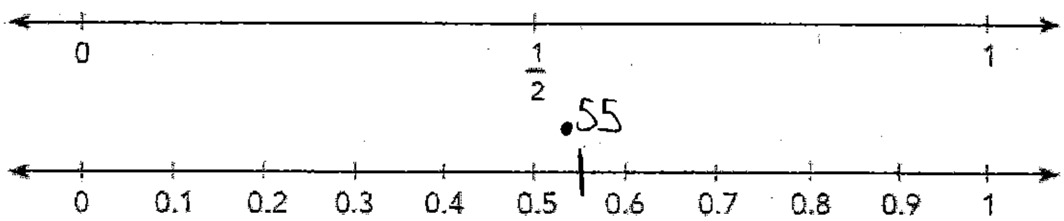
18. $14.4 \boxed{>} 13.38$

19. $8\frac{5}{12} \boxed{>} \frac{100}{12} = 8\frac{4}{12}$

Guided Practice

Complete.

20. Which is greater, $\frac{1}{2}$ or 0.55?



$\frac{1}{2} =$ <u>.5</u>

<u>.5</u> lies to the <u>left</u> of 0.55.

So, $0.55 \boxed{>} \frac{1}{2}$.

Compare each $\boxed{}$ using > or <. Use a number line to help you.

21. $0.2 \boxed{<} \frac{1}{4} = .25$

22. $\frac{3}{4} \boxed{>} 0.7$
$.75$

23. $0.89 \boxed{>} \frac{4}{5} = \frac{80}{100} = .80$

24. $0.25 \boxed{>} \frac{1}{5} = \frac{2}{10} = .2$

25. $\frac{2}{5} \boxed{>} 0.3$
$\frac{4}{10} = .4$

26. $3.26 \boxed{<} 3\frac{5}{8}$ 3.6 $8\overline{)5.0}$ .6
-48
20

1.2 – Prime Factorization

factor - a whole number that **divides evenly** into a whole number

composite number - a counting number that has **more** than **two factors**

prime number - a counting number that has **exactly two** different **factors 1 and itself**

Divisibility Rules

A whole number is divisible by:

2 if the last digit is 0, 2, 4, 6, or 8. even

3 if the sum of its digits is divisible by 3. 513 5+1+3=9 9÷3=3

4 if number formed by the last 2 digits is divisible by 4. 724 24÷4=6

5 if the last digit is 5 or 0. 780 or 785

6 if the number is divisible by both 2 and 3.

8 if the number formed by the last 3 digits is divisible by 8.

9 if the sum of the digits is divisible by 9. 7,245 7+2+4+5=18 18÷9

10 if the last digit is 0.

Learn Identify composite numbers.

Another way to represent a whole number is to write it as a product of its **factors**.

Find all the factors of 18.

$18 = 1 \times \boxed{18}$ $18 = 2 \times \boxed{9}$ $18 = 3 \times \boxed{6}$

The factors of 18 are $\boxed{1, 2, 3, 6, 9, 18}$

The number 18 is an example of a composite number.

A composite number has more than two different whole-number factors.

18 has six factors, so it is a composite number.

The number 3 is an example of a **prime number**. A prime number has only two factors, the number itself and 1.

Learn Write a composite number as a product of its prime factors.

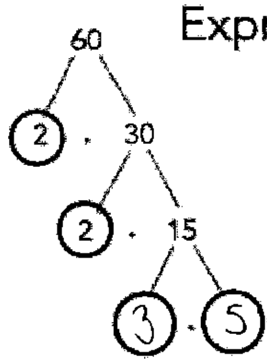
Express 60 as a product of its prime factors.

Method 1 Factor Ladder

2	60	Divide by prime factor 2.
2	30	Divide by prime factor 2.
3	15	Divide by prime factor 3.
	5	

Start dividing the number by its least prime factor. Continue dividing until the quotient is a prime number.

Method 2



Express 60 as a product of its prime factors.

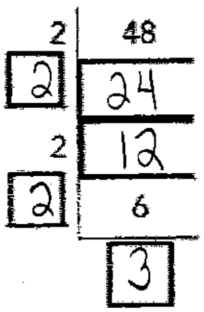
$60 = \boxed{2} \cdot \boxed{2} \cdot \boxed{3} \cdot \boxed{5}$

Guided Practice

Complete.

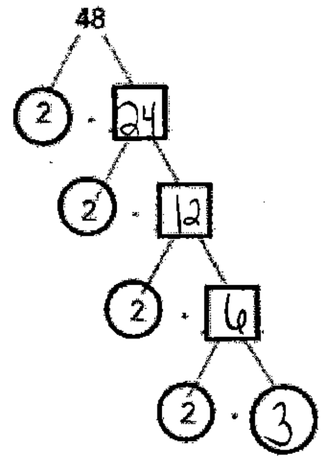
1 Express 48 as a product of its prime factors.

Method 1



$48 = 2 \times \boxed{2} \times 2 \times \boxed{2} \times \boxed{3}$

Method 2



$48 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot \boxed{3}$

1.3 – Common Factors and Multiples

common factor - a number that is a factor of two or more whole numbers

greatest common factor - the common factor of two or more numbers that has the greatest value

Factors are few.

Guided Practice Factor T

Find the common factors of each pair of numbers.

2. 16 and 24 1, 2, 4, 8

3. 27 and 35 1

4. 36 and 50 1, 2

5. 40 and 54 1, 2

16	
1	16
2	8
4	4

24	
1	24
2	12
3	8
4	6

36	
1	36
2	18
3	12
4	9
6	6

50	
1	50
2	25
5	10

27	
1	27
3	9

35	
1	35
5	7

40	
1	40
2	20
4	10
5	8

54	
1	54
2	17
3	18
6	9

Guided Practice

Complete.

6 Find the greatest common factor of 20 and 32.

Method 1

20		32	
1	20	1	32
2	10	2	16
4	5	4	8

Method 2 **Multiply only the common factors.**

By prime factorization

20: 2, 10 → 2, 5
32: 4, 8 → 4 → 2, 2; 8 → 4, 2 → 4 → 2, 2

$20 = 2 \cdot 2 \cdot 5$
$32 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
$GCF = 2 \cdot 2$
$= 4$

Method 3
Factor Ladder

$$\begin{array}{r|ll} 2 & 20, & 32 \\ \boxed{2} & \boxed{10}, & \boxed{16} \\ & 5, & 8 \end{array}$$

$2 \times \boxed{2} = \boxed{4}$ The greatest common factor of 20 and 32 is 4

Show your work. You may use any method(s) you prefer.

Find the greatest common factor of each pair of numbers.

7 15 and 27 = 3

8 36 and 54 = 18

9 48 and 72 = 24

10 40 and 100

15: 3, 5
27: 9, 3 → 9 → 3, 3

$15 = 3 \cdot 5$
$27 = 3 \cdot 3 \cdot 3$
$GCF = 3$

36: 6, 6 → 2, 3, 2, 3
54: 6, 9 → 2, 3, 3, 3

$36 = 2 \cdot 2 \cdot 3 \cdot 3$
$54 = 2 \cdot 3 \cdot 3 \cdot 3$
$GCF = 2 \cdot 3 \cdot 3$
$= 2 \cdot 9$
$= 18$

48: 6, 8 → 2, 3, 4, 2 → 4 → 2, 2
72: 9, 8 → 3, 3, 2, 4 → 4 → 2, 2

$48 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$
$72 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$
$GCF = 2 \cdot 2 \cdot 2 \cdot 3$
$= 8 \cdot 3$
$= 24$

40: 4, 10 → 2, 2, 2, 5
100: 10, 10 → 2, 5, 2, 5

$40 = 2 \cdot 2 \cdot 2 \cdot 5$
$100 = 2 \cdot 2 \cdot 5 \cdot 5$
$GCF = 2 \cdot 2 \cdot 5$
$= 4 \cdot 5$
$= 20$

Guided Practice

Complete.

11. Express 18 + 45 as a product of the greatest common factor of the numbers and another sum.

By prime factorization,

18
9 2
3 3

45
9 5
3 3

18 = $2 \cdot 3 \cdot 3$

45 = $3 \cdot 3 \cdot 5$

GCF = $3 \cdot 3$
9

Greatest common factor of 18 and 45 = $\boxed{3} \cdot \boxed{3}$

Distributive Property $= \boxed{9}$

$18 + 45 = \boxed{9} \cdot \boxed{2} + \boxed{9} \cdot \boxed{5}$

$= \boxed{9}(\boxed{2} + \boxed{5})$

Distributive Property

Express the sum of each pair of numbers as a product of the greatest common factor of the numbers and another sum.

12. 35 + 91

35
7 5

91
7 13

$7(5+13)$

13. 60 + 85

60
3 20
4 5
2 2

85
5 17

$5(2 \cdot 2 \cdot 3 + 17)$

$5(12+17)$

14. 24 + 64

24
4 6
2 2 2 3

64
8 8
4 2 4 2
2 2 2 2

$24 = 2 \cdot 2 \cdot 2 \cdot 3$

$64 = 2 \cdot 2 \cdot 2 \quad \cdot 2 \cdot 2 \cdot 2$

$GCF = 2 \cdot 2 \cdot 2 = 8$

$8(3 + 2 \cdot 2 \cdot 2)$

$8(3+8)$

common multiple - a number that is a multiple of two or more whole numbers

least common multiple - the common multiple of two or more numbers that has the least value

Multiples are many ...

Guided Practice

Complete. Skip counting

15 Find the first three common multiples of 3 and 5.

The multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45

The multiples of 5 are 5, 10, 15, 20, 25, 30, 35, 40, 45, 50

The first three common multiples of 3 and 5 are $\boxed{15}$, $\boxed{30}$, and $\boxed{45}$

List the first ten multiples of each pair of numbers. Then find the common multiples of each pair of numbers from the first ten multiples.

16 6 and 12

6, 12, 18, 24, 30, 36, 42, 48, 54, 60

12, 24, 36, 48, 60, 72, 84, 96, 108, 120

common: 12, 24, 36, 48, 60

17 7 and 11

7, 14, 21, 28, 35, 42, 49, 56, 63, 70

11, 22, 33, 44, 55, 66, 11, 77, 88, 99, 110

Common: none

Guided Practice

Complete.

18 Find the least common multiple of 8 and 10.

Method 1

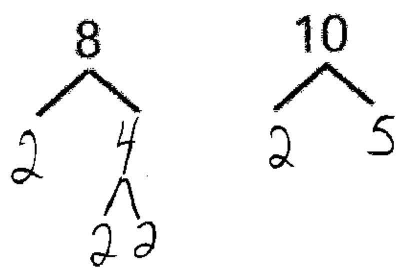
multiples of 8, 16, 24, 32, 40, 48

multiples of 10, 20, 30, 40

least common multiple 40

Method 2

By prime factorization



$8 = 2 \cdot \boxed{2} \cdot \boxed{2}$

$10 = 2 \cdot \boxed{5}$

Least common multiple

$= 2 \cdot \boxed{2} \cdot \boxed{2} \cdot \boxed{5}$

$= \boxed{40}$

Method 3

$$\begin{array}{r|ll} 2 & 8, & 10 \\ \hline & \boxed{4}, & \boxed{5} \end{array}$$

$2 \times \boxed{4} \times \boxed{5} = \boxed{40}$ least common multiple

Show your work. You may use any method(s) you prefer.

Find the least common multiple of each pair of numbers.

19 3 and 7

$3 \cdot 7$

21

$LCM = 21$

20 5 and 12

5 12

2 6

$5 = 5$

$12 = 2 \cdot 6$

$LCM = 5 \cdot 2 \cdot 6$

$= 10 \cdot 6$

$= 60$

21 4 and 9

4 9

2 2 3 3

$4 = 2 \cdot 2$

$9 = 3 \cdot 3$

$LCM = 2 \cdot 2 \cdot 3 \cdot 3$

$= 4 \cdot 9$

$= 36$

22 6 and 11

6 11

2 3

$6 = 2 \cdot 3$

$11 = 11$

$LCM = 2 \cdot 3 \cdot 11$

$= 6 \cdot 11$

$= 66$

1.4 – Square and Square Roots

exponent

base $\rightarrow$ 8^2 Area of the square = edge • edge = e^2

e

e e

e

perfect square - the square of a whole number

Guided Practice

Find the square of each number.

1. 2 $2 \cdot 2 = 4$
2. 6 $6 \cdot 6 = 36$
3. 9 $9 \cdot 9 = 81$
4. 11 $11 \cdot 11 = 121$

b) Find the square root of 100.

Method 1

Recalling the multiplication facts of 10, you know that

$\boxed{10} \times \boxed{10} = 100$

So, $\sqrt{100} = \boxed{10}$

Method 2

By prime factorization

100

2 50

2 25

5 5

Must end with 2 pairs of prime numbers.

$$100 = \boxed{2}\cdot\boxed{2}\cdot\boxed{5}\cdot\boxed{5}$$ Write the prime factorization

$$= (\boxed{2\cdot 5})\cdot(\boxed{2\cdot 5})$$ Apply the Commutative Property of Multiplication

$$= (\boxed{2\cdot 5})^2$$ Rewrite using an exponent

$$= \boxed{10}^2$$

So, $\sqrt{100} = \boxed{10}$

Show your work. You may use any method(s) you prefer.

Guided Practice

Find the square root of each number.

5. 25 5

6. 64 8

7. 144 = 12

8. 196 14

25

5 5

5^2

$\sqrt{25} = 5$

144

2 72

2 36

6 6

$(2\cdot 6)^2$

12^2

64

4 16

4 4

2 2

$(2\cdot 4)^2$

8^2

196

2 98

2 49

7 7

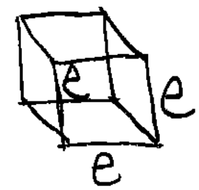
$(2\cdot 7)^2$

14^2

1.5 – Cube and Cube Roots

Volume of cube = edge • edge • edge $= e^3$

length • width • height



perfect Cube - the ~~square~~ cube of a whole number

Guided Practice

Find the cube of each number.

1 5

$5 \cdot 5 = 25 \cdot 5$

$= 125$

2 6

$6 \cdot 6 = 36 \cdot 6$

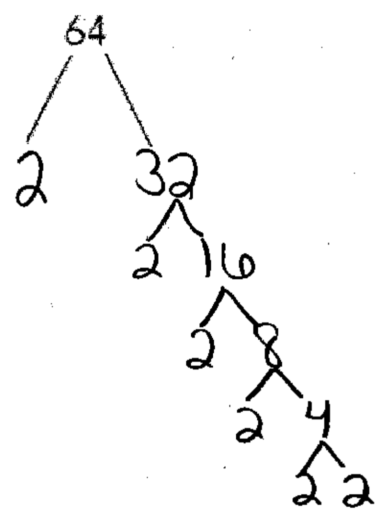
$= 216$

3 9

$9 \cdot 9 = 81 \cdot 9$

$= 729$

b) Find the cube root of 64.



By prime factorization,

Must end with 3 identical pairs of prime numbers.

$64 = \boxed{2} \cdot \boxed{2} \cdot \boxed{2} \cdot \boxed{2} \cdot \boxed{2} \cdot \boxed{2}$ Write the prime factorization.

$= (\boxed{2 \cdot 2}) \cdot (\boxed{2 \cdot 2}) \cdot (\boxed{2 \cdot 2})$ Write parentheses.

$= (\boxed{2 \cdot 2})^3$ Rewrite using an exponent.

$= \boxed{4}$

So, $\sqrt[3]{64} = \boxed{4}$

Show your work. You may use any method(s) you prefer.

Guided Practice

Find the cube root of each number.

4 216

216
2 108
2 54
2 27
3 9
3 3

$(2 \cdot 3)^3$
6^3

5 343

343
7 49
7 7

7^3

6 1,000

1000
10 100
10 10

10^3

Order of Operations

STEP 1 Evaluate inside $\boxed{\text{parentheses}}$

STEP 2 Evaluate $\boxed{\text{exponents}}$

STEP 3 $\boxed{\text{Multiply}}$ and $\boxed{\text{divide}}$ from $\boxed{\text{left to right.}}$

STEP 4 $\boxed{\text{Add}}$ and $\boxed{\text{subtract}}$ from $\boxed{\text{left to right.}}$

Guided Practice

Complete.

7 Find the values of $5^2 + 5^3$ and $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$.

25 + 125

150

25 · 125

3,125

$$\begin{array}{r} 125 \\ \times 25 \\ \hline 625 \\ 250 \\ \hline 3,125 \end{array}$$

8 $6^3 + 4^2$

6·6·6 + 4·4

36·6 + 16

216 + 16

232

9 $7^3 - 4^3$

7·7·7 − 4·4·4

49·7 − 16·4

343 − 64

279

10 $3^2 \times 5^3 + 9^2$

3·3 · 5·5·5 + 9·9

9 · 125 + 81

1,125 + 81

1,206

11 $8^3 \div 4^2 - 5^2$

8·8·8 ÷ 4·4 − 5·5

64·8 ÷ 16 − 25

512 ÷ 16 − 25

32 − 25

7

12 $7^2 + 6^3 \div 2^3$

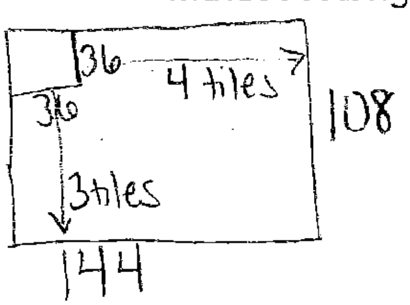
$7 \cdot 7 + 6 \cdot 6 \cdot 6 \div 2 \cdot 2 \cdot 2$
$49 + 216 \div 8$
$49 + 27$
76

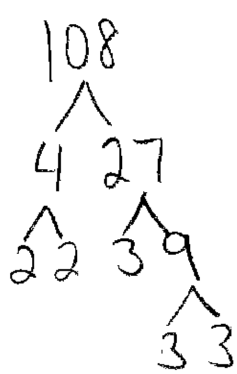
13 $9^3 - 4^2 \times 3^3$

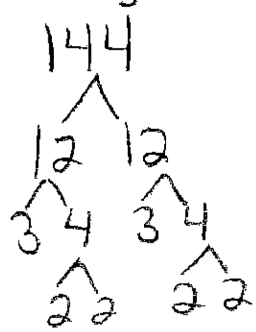
$9 \cdot 9 \cdot 9 - 4 \cdot 4 \cdot 3 \cdot 3 \cdot 3$
$729 - 16 \cdot 27$
$729 - 432$
297

Brain @ Work

Mr. Henderson wants to tile his patio that is rectangular in shape. His patio measures 108 inches by 144 inches. Find the fewest square tiles he can use without cutting any of them. (Hint: First find the largest size tile he can use.)







$108 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$
$144 = 2 \cdot 2 \cdot 3 \cdot 3$ 2·6
$GCF = 2 \cdot 2 \cdot 3 \cdot 3$
$= 4 \cdot 9$
$= 36$

$108 \div 36 = 3$
$144 \div 36 = 4$
$3 \cdot 4 = 12$

The fewest square tiles he can use is 12.